General Physics: Electromagnetism, Problem Set 9

Exercise 1 :

A long straight wire of radius a carries a current that is uniformly distributed over its cross-section. Find the magnetic field both inside and outside the wire.

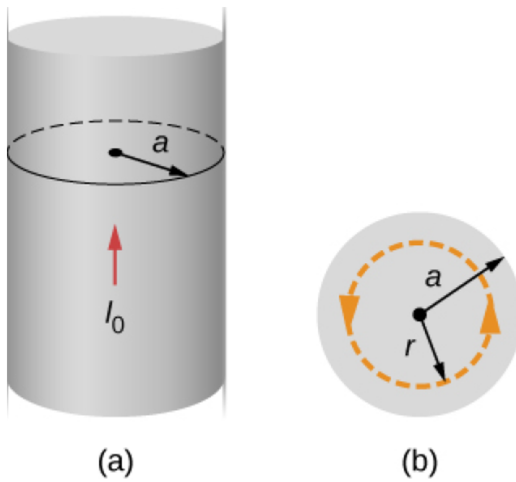


Figure 1: (a) A model of a current-carrying wire of radius a and current I_0 (b) A cross-section of the same wire showing the radius a and the Ampere's loop of radius r.

Exercise 2 :

Consider an infinitely long, cylindrical conductor of radius R carrying a current I with a non-uniform current density

$$J = \alpha r \tag{1}$$

where α is a constant. Find the magnetic field everywhere.

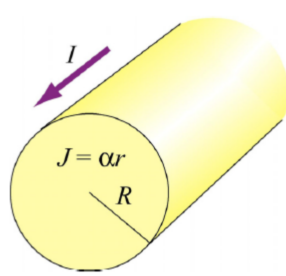
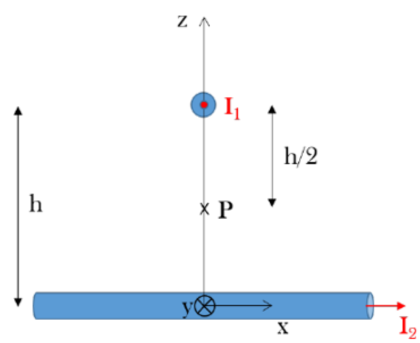


Figure 2: Non-uniform current density

Exercise 3 :

Two long wires of radius a are perpendicularly oriented as shown in figure below. The upper wire has a current I_1 in the $\hat{y}$ direction and the lower cable has a current I_2 in the $\hat{x}$ direction.



(a) Find the magnetic field along the z axis, between $z = a$ and $z = h$.

(b) For $I_1 = 100$ A and $I_2 = 150$ A, with the distance $h = 2.5$ cm, what is the magnitude of the magnetic field at point P?

(c) Describe the direction of the compass needle placed in point P.
Hint: The magnetic field of Earth is about $5 \cdot 10^{-5}$ T.

Exercise 4 :

Consider two infinitely long wires carrying currents are in the x direction.

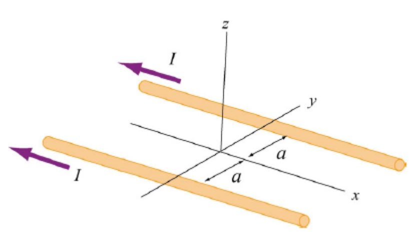


Figure 3: Non-uniform current density

(a) Draw a schematic of the magnetic field pattern in the yz-plane.

(b) Find the distance d along the z-axis where the magnetic field is maximum.

Exercise 5 :

Consider a toroid as shown in Figure 4 with a big radius R and a small radius a. The toroid contains N turns of the wire with current I. Suppose that the number of turns N is very large, such that we can consider cylindrical symmetry. Calculate the magnetic field $\overrightarrow{B}$ for:

(a) $r < R - a$;

(b) $R - a < r < R + a$;

(c) $r > R + a$;

and draw the lines of the field. What is the magnitude of the magnetic field in $r = R$ if $I = 500$ A, $N = 100$ and $R = 0.5$ m?

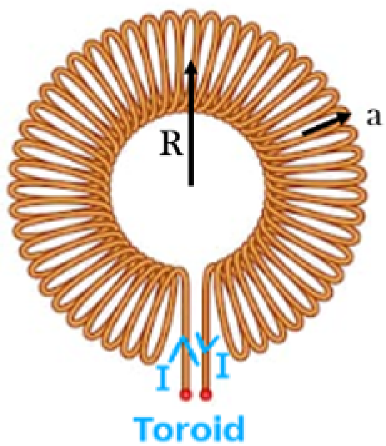


Figure 4: Toroid

Exercise 6 :

Find the minimum diameter of the wires d that can transmit $P = 225$ MW of electricity with only a 2.0% loss. Their length is $l = 185$ km. Assume there are two wires to make a complete circuit (the length is thus doubled). The wires are to be made of aluminum ($\rho = 2.6 \cdot 10^{-8}\ \Omega \cdot m$) and the voltage is $V = 660$ kV.